SIMULTANEOUS DETERMINATION OF THERMOPHYSICAL CHARACTERISTICS OF MOIST MATERIALS FOR ANY VALUES OF THE FOURIER NUMBER

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An improved method of simultaneous determination of the thermophysical characteristics of moist materials, applicable at any (small or large) values of the Fourier number, is discussed.

In a number of cases, combined methods of determining several thermophysical characteristics using a standard substance are very effective and convenient [1–3]. One of these is the method of N. N. Bab'ev, which is used to determine five coefficients (λ, a, c, a_m, δ) in one experiment and does not involve cutting up the sample. A disadvantage of the method is that it is useful for determing the heat transfer coefficients only at small values of the Fourier number (Fo $\leq$ $\leq$ 0.5).

The method of N. N. Bab'ev involves a system consisting of a finite (test) and an infinite (standard) cylinder butted together at the ends and thermally insulated on the lateral surfaces. The free end of the finite cylinder is heated by a constant heat source. The basis of this approach is the analytical solution of problems of heat conduction by an operational method using an expression for representing the unknown function (temperature) in a form convenient for practical use at Fo $\leq$ 0.5, i.e.,

$$T(x,\ S) = A_1 \exp(\sqrt{S/a}\,x) + B_1 \exp(\sqrt{S/a}\,x). \qquad (1a)$$

We shall investigate the possibility of solving the problem in a form convenient for Fo > 0.5. In this case the solution must be sought in the form

$$T(x,\ S) = A \coth \sqrt{S/a}\,x + B \sinh \sqrt{S/a}\,x. \qquad (1b)$$

Then, on transforming to the original, for the conditions of the problem we obtain

$$\cot \mu_n = K_\varepsilon i; \quad i = \sqrt{-1}.$$

Since the quantity K_ε is always positive, the equation has only imaginary roots μ_n, and to express the original by means of functions of real variables is impossible. This explains the inapplicability of the given method for values of Fo > 0.5.

Our object is to modify the initial data so as to obtain real solutions in a form convenient for large values of Fo. Accordingly, we shall examine a system of two finite cylinders (test and standard) of similar length, touching at the ends and thermally insulated on their lateral surfaces. The free ends of the cylinders are heated by constant heat fluxes of the same magnitude. The solution of the problem using equation (1b) leads to the characteristic equation

$$K_\varepsilon \tan \mu_n + \mathrm{tg}\, K_a^{1/2} \mu_n = 0$$

with real roots μ_n.

Assuming, as usual, that at Fo $\geq$ 0.5 exponential functions of the form exp ($-k_a \mu_n^2$ Fo) are equal to zero, after simplification we obtain the following expression for the temperature of the test (t_1) and standard (t_2) specimens:

$$t_1(x,\ \tau) - t_0 = \frac{2q_c K_\lambda}{\lambda_1 (K_\lambda + K_a)} \left\{ \frac{a_1 \tau}{R} + \frac{x^2 + R(1 - K_\lambda^{-1} K_a)x}{2R} + \frac{(K_a^2 - 3K_\lambda K_a + K_\lambda - 3K_a)R}{12(K_\lambda + K_a)} \right\}, \qquad (2a)$$

$$t_2(x,\ \tau) - t_0 = \frac{2q_c K_a}{\lambda_2 (K_\lambda + K_a)} \left\{ \frac{a_2 \tau}{R} + \frac{x^2 + R(K_\lambda K_a^{-1} - 1)x}{2R} + \frac{(K_a - 3K_\lambda + K_a^{-1} K_\lambda - 3)R}{12(K_\lambda + K_a)} \right. . \qquad (2b)$$

From the solutions (1) and (2) the values of a_1 and λ_1 can be determined. For this we consider the following excess temperature ratios determined from experiment:

$$\frac{t_1(0,\ \tau''') - t_0}{t_1(0,\ \tau'') - t_0} = \frac{t_2(0,\ \tau''') - t_0}{t_2(0,\ \tau'') - t_0}, \qquad (3)$$

$$\frac{t_1(-R,\ \tau) - t_0}{t_1(0,\ \tau) - t_0} = \vartheta_1, \qquad (4)$$

$$\frac{t_2(R,\ \tau) - t_0}{t_2(0,\ \tau) - t_0} = \vartheta_2. \qquad (5)$$

The unknown value of a_1 is obtained from the simultaneous solution of (3) and (5):

$$a_1 = \frac{K_\lambda (1 - m) R^2}{2(1 - \vartheta_2)[(\tau''' - m\tau'') - (1 - m)\tau]}. \qquad (6)$$

The number K_λ is determined from the simultaneous solution of (1a), (3), (4), and (6):

$$K_\lambda = \{4a_2 (1 - \vartheta_2)[(\tau''' - m\tau'') - (1 - m)\tau] - (1 - m)R^2\} \times \{2a_2 (1 - \vartheta_1)[(\tau''' - m\tau'') - (1 - m)\tau]\}^{-1}. \qquad (7)$$

In general, the value of τ may lie in the range $\tau'' \leq$ $\leq \tau \leq \tau'''$. From the conditions of averaging we shall henceforth take

$$\tau = 0.5(\tau''' + \tau''). \qquad (8)$$

The unknown values of λ_1 and C_1 are determined from the formulas $\lambda_1 = K_\lambda \lambda_2$; $c_1 = \lambda_1 / a_1 \rho_1$.

The thermal gradient coefficient δ is determined from the basic equation of steady-state methods:

$$\frac{\partial u}{\partial x} = -\delta \frac{\partial t}{\partial x}. \qquad (9)$$

We will first clarify the character of the curves representing the temperature distribution along the length of the cylinders Fo ≥ 0.5.

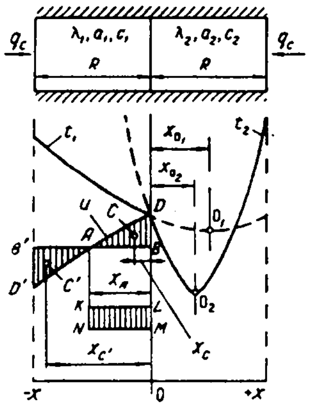

Fig. 1. Distribution of temperature and moisture content along the length of the samples.

Analysis of the solutions (1a) and (1b) shows that these curves are parabolas unsymmetrical with respect to the ordinate axis (Fig. 1). The abscissas of the vertices of these parabolas are

$$x_{0_1} = -R(K_\lambda K_a^{-1} - 1)/2K_\lambda K_a^{-1};\quad x_{0_2} = -R(K_\lambda K_a^{-1} - 1)/2.$$

Consequently, two cases are possible:

$$1)\quad K_\lambda K_a^{-1} > 1;\quad x_{0_1} < 0;\quad x_{0_2} < 0; \tag{10}$$

$$2)\quad K_\lambda K_a^{-1} < 1;\quad x_{0_1} > 0;\quad x_{0_2} > 0. \tag{11}$$

In the first case the vertices of the parabolas lie in the region of the test sample, and in the second in the region of the standard. With respect to convenience of analysis of the experimental data, it is necessary to consider the case

$$K_\lambda K_a^{-1} = c_2\rho_2/c_1\rho_1 > 1 \quad \text{or} \quad c_2\rho_2 > c_1\rho_1. \tag{12}$$

Thus, the volume heat capacity of the standard must be greater than that of the test sample.

In determining the properties of many fibrous materials, dry quartz sand was taken as a standard.

Starting from (9), we assume that in the quasi-steady state a parabolic temperature distribution corresponds to a parabolic moisture content distribution. On a section of the test sample the equations of these parabolas can be written in the form

$$a_t x^2 - b_t x = t(x, \tau) - t(0, \tau), \tag{13a}$$

$$-a_u x^2 + b_u x = -u(0, \tau) + u(x, \tau). \tag{13b}$$

Hence it follows that $a_u = \delta a_t$, $b_u = \delta b_t$.

We express the unknown coefficient as

$$\sigma = a_u/a_t.$$

The coefficient a_u can be determined from the following considerations. The mean moisture content of the test sample is assumed constant, since it is hermetically sealed and insulated from moisture. Under the action of a temperature field redistribution of moisture takes place inside the test sample, leading to disturbance of the equilibrium of the system. From experiment it is possible to determine the increment Δm needed to restore the system to equilibrium. Starting from (13b), by the methods of geometrical statics we find the abscissa of the point A and the centers of gravity C' and C of the corresponding to figures(Fig. 1):

$$x_A = -\left\{3\left[3\frac{b_u^2}{a_u^2} + 2R\left(3\frac{b_u}{a_u} + 2R\right)\right] + 3\frac{b_u}{a_u}\right\}^{1/2} \Big/ 6, \tag{14}$$

$$x_{C'} = \frac{3(x_A^4 + R^4) - 2\frac{b_u}{a_u}(x_A^3 - 2R^3) - 6x_A^2 R^2 + 6\frac{b_u}{a_u}x_A R^2}{2\left[2(2x_A^3 - R^3) - 3\frac{b_u}{a_u}(x_A^2 + R^2) + 6x_A^2 R - 6\frac{b_u}{a_u}x_A R\right]}, \tag{15}$$

$$x_C = \frac{x_A(3x_A - 2b_u/a_u)}{2(4x_A - 3b_u/a_u)}. \tag{16}$$

The ratio b_u/a_u is found from the solution of (1):

$$\frac{b_u}{a_u} = \frac{b_t}{a_t} = \frac{R(K_\lambda - K_a)}{K_\lambda}. \tag{17}$$

This ratio can be represented in another form, by considering equidimensional figures KLMN and ADB. From the equality of the areas of these figures we have

$$-\left(-a_u\frac{x_A^3}{3} + b_u\frac{x_A^2}{2}\right) + \left(-a_u x_A^3 + b_u x_A^2\right) = \Delta M\frac{R}{M_{\text{dry}}},$$

whence

$$\frac{b_u}{a_u} = \frac{2(2x_A^3 + 3\Delta MR/a_u M_{\text{dry}})}{3x_A^2}. \tag{18}$$

The quantity ΔM can be calculated from the value Δm and the geometrical characteristics

$$\Delta M = 2\Delta mR/(x_{C'} - x_C).$$

Equating the right hand sides of (17) and (18), we obtain

$$a_m = \frac{-6\Delta MK_\lambda R}{M_{\text{dry}}x_A^2[4K_\lambda x_A - 3R(K_\lambda - K_a)]}. \tag{19}$$

To calculate the coefficient a_t we take (13a), putting $x = -R$:

$$a_t R^2 + b_t R = \Delta t. \tag{20}$$

From (20), with account for (17), we obtain

$$a_t = K_\lambda \Delta t / R^2 (2K_\lambda + K_a)$$

from which we get, in final form,

$$\delta = \frac{a_u}{a_t} = \frac{6\Delta M (2K_\lambda - K_a) R^3}{x_A^2 [4K_\lambda x_A - 3R (K_\lambda - K_a) \Delta t M_{\mathrm{dry}}]}. \quad (21)$$

The coefficient a_m is determined using

$$a_m = i \left[\rho_0 \left(\delta \frac{\partial t}{\partial x} + \frac{\partial u}{\partial x} \right) \right]^{-1}, \quad (22)$$

where

$$i = \frac{\Delta(\Delta M)}{S \Delta\tau} = \frac{4R(\Delta m'' - \Delta m')}{S(\tau'' - \tau')}.$$

It is important that this interval of time $\Delta\tau$ must precede and immediately adjoin the moment corresponding to onset of the quasi-steady state. In practice we determined the value of τ'' from the cessation of the moisture flow when the equilibrium of the system was not disturbed. This was further checked against the emergence of a linear dependence of the change of temperature with time.

Numerical values of $\partial t/\partial x$ and $\partial u/\partial x$ are found from (13a) and (13b) for a value of the time corresponding to the condition $\tau = 0.5\ (\tau' + \tau'')$. We established by direct experiment that in a given time interval $\Delta\tau = \tau'' - \tau'$ a parabolic law of distribution temperature and moisture gives a better approximation than a linear law. The developed form of (22) is

$$a_m = i \{ \rho_0 [\delta (2a_t x - b_t) - 2a_u x + b_u] \}^{-1}$$
$$\text{at} \quad x = x_A.$$

The complexity of (7), (14), (15) and (21) is only apparent. In practice the computation takes a few minutes.

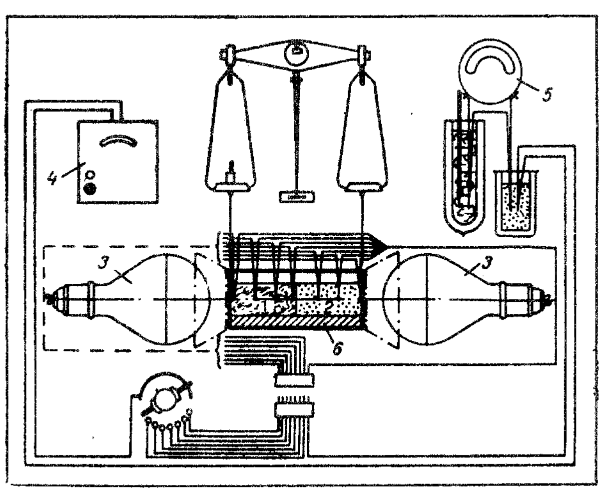

Fig. 2. Schematic of experimental setup: 1) green corn (test sample); 2) quartz sand (standard); 3) infrared lamp; 4) PP potentiometer; 5) galvanometer; 6) thermal insulation.

The experimental setup is shown schematically in Fig. 2. The samples were placed in a glass cylinder 36 mm in diameter and 150 mm in length. In the middle and at the ends of the cylinder copper foil discs were placed to give a thorough hermetic seal. The temperature of the samples was measured at the indicated points by copper-constantan thermocouples. The "cold" junction was placed in the standard substance at the temperature of the surrounding medium, which was checked by the galvanometer. Consequently, the readings of the potentiometer gave the excess temperature. All the experimentally determined quantities were measured every 3–5 min.

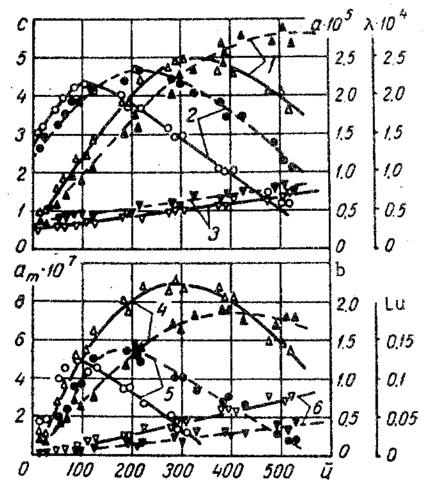

Fig. 3. Dependence of thermophysical characteristics of green corn on moisture content, %: 1) λ; 2) a; 3) c; 4) a_m; 5) δ; 6) Lu; the continuous lines correspond to samples obtained by chopping at right angles to the fibers; the broken lines to crushed samples.

The apparatus is so designed that Babev's method and our proposed method can be combined in one experiment. In this case at first only the left lamp was switched on, then after the temperature at the right free end began to rise, the right lamp. Figure 3 shows the results of determining the coefficients of heat and mass transfer by this and other methods.

As test material we took green corn of the Odessa 10 variety in the state of milky-waxy ripeness. Test samples were prepared from ears of different moisture contents chopped at right angles to the fibers (distance between chops 30–40 mm) and crushed between rollers. Comparison of the curves shows that crushing significantly changes nearly all the thermophysical characteristics (apart from the specific heat) both in absolute magnitude and in the form of their dependence on moisture content. The general nature of these changes corresponds to a "weakening" of the bond between moisture and material, to an increase in the quantity of free water. In particular, the points of inflection of the λ and a curves (Fig. 3) are shifted to the left. The same shift is observable in the curves of both a_m and δ. In this case the numerical values of a_m in the range of moisture contents from 50 up to 400% are considerably higher for the crushed corn. The maximum value of the thermal gradient coefficient δ is reduced by crushing. The value of δ approaches zero at a lower moisture content (of the order of 300–350%) than for chopped corn. The moisture content corresponding to a zero value of δ is, as is known, the approximate boundary between adsorption-bound and free moisture. This in some measure confirms the assumed increase in the quantity of free moisture on crushing. A further indication is the steeper slope of the line characterizing the change in the Lu number for the crushed corn.

As our direct experiments show [4], crushing the green corn significantly shortens the drying time and sharply increases the quality of the dried product.

NOTATION

$K_a = \frac{a_1}{a_2}$; $K_\lambda = \frac{\lambda_1}{\lambda_2}$; $K_\varepsilon = \frac{\lambda_1}{\lambda_2}\sqrt{\frac{a_2}{a_1}}$) coefficients characterizing the thermal properties of the first cylinder in relation to the second; c) specific heat; a_m) mass-transfer coefficient of potential conduction; t_0) temperature of surrounding medium; Δt) temperature difference of ends of test sample determined by experiment; q_c) heat flux density; R) length of cylinder; m) ratio of excess temperatures of abutting ends of cylinders at times τ''' and τ''; τ'') time corresponding to beginning of quasi-steady state; ϑ) ratio of excess temperatures of heated and abutting ends of cylinders at time τ; S) cross-sectional area of cylinders; ΔM) mass of displaced moisture; M_{dry}) dry mass of sample; ρ_0) density of dry sample; i) mositure flux; Δm) mass increment restoring system to equilibrium; Lu) Luikov number; Fo) Fourier number. Indices: 1) test sample; 2) standard.

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